**Goal:** Unit Resolution (f, c) -> Logical Entailment (f, c)

*If c can be derived from f with Unit Resolution, then c must be logically entailed from f.*

**Proof Description:**

Induction on unit res: 2 cases

Subsumption Case:

Unfolding Definitions: We expand the definitions of “entails”, "models\_formula" and "models\_clause".

Assumptions:

We assume the existence of a model "H\_model\_clause" that satisfies the "clause".

"h\_model\_clause\_prop" represents the property that the model "H\_model\_clause" satisfies clause "c".

"h\_model\_prop" is a proposition derived from "h\_model\_clause\_prop" and is used to establish the entailment of "c".

"h\_model\_clause\_prop2" is another property related to the model "H\_model\_clause", which may be relevant for further reasoning within the subsumption case.

Application of Hypothesis: We specialize "h\_model\_prop" with clause "c".

Specialization: We specialize "h\_model\_prop" with arguments "h\_model\_clause\_prop", "H", and "h\_model\_clause\_prop" (named as "h\_model\_prop\_specialized").

Assumptions: We introduce variables "l0" and "l0prop" to represent certain properties.

Destructing: We use the destruct tactic to examine the properties of the clause "c" and "c2" under the given model assumptions.

Application of Proposition: We apply the derived proposition.

Existential Introduction: We introduce another existential variable "x".

Application of Proposition: We apply the derived proposition.

Conclusion: Thus, "f" entails "c".

Resolution Case:

Application of Lemma:

We apply the lemma: "f entails c" and "f entails neg l" imply "f entails c \ l".

This lemma helps establish the entailment of "f" to the result clause "remove\_literal\_from\_clause l c".

Prove the Goal:

Given the induction hypotheses IHunitres\_c1\_1 and IHunitres\_c1\_2, which state that for any model "m" satisfying certain properties, "f" entails "c" and "f" entails "c2", respectively:

We need to show that for any such model "m" satisfying the properties:

If "f" entails "c" and "f" entails "c2", then "f" entails "remove\_literal\_from\_clause l c" (the goal clause).

To achieve this, we use the result of the lemma and the given induction hypotheses:

We know that "f" entails "c" (from IHunitres\_c1\_1) and "f" entails "c2" (from IHunitres\_c1\_2).

By applying the lemma, we conclude that "f" entails "c \ l".

Thus, "f" entails "remove\_literal\_from\_clause l c", which completes the proof of the goal.

This completes the proof of the goal.

Conclusion: Therefore, if "unitres f c" holds, then "f" entails "c".